MID-SEMESTER EXAMINATION
M. MATH I YEAR, II SEMESTER 2015-2016
COMPLEX ANALYSIS

Max. Score:100 Time limit: 3hrs.

1. Find all positive integers $n$ such that the map $z \to z^n$ maps the upper half plane into itself. [10]

2. Let $f(z) = \exp(\exp(\exp(z)))$. Show that $|f(z)| \le e^e$ if $|z| \le 1$. [10]

3. Find the largest open set on which $\int\limits_0^\infty \frac{e^{tz}}{1+t^2} dt$ is defined and analytic. [15]

4. Find the nature of singularity of $e^{\frac{1}{\sin z}}$ at 0. Justify your answer. [10]

5.

a) Let $n$ be a positive integer $> 1$. Give an example of a holomorphic function on $\mathbb{C}\backslash\{0\}$ which has a holomorphic $m-th$ root for $m = n-1$ but not for $m = n$. [20]

b) Let $f$ be a holomorphic in $\mathbb{C}\backslash\{0\}$ and $f^n(z) = z^m$ for all $z$ where $m$ and $n$ are positive integers. Show that $n$ divides $m$. [10]

6. Show that there is no holomorphic function $f$ on the open unit disc $U$ such that $|f(z)| \to \infty$ as $|z| \to 1$. [10]

7. Let $f$ be a conformal equivalence of the open unit disc $U$ onto itself. If $f$ has more than one fixed point show that $f(z) = z$ for all $z \in U$. [15]

Hint: apply Schwartz Lemma to $\phi_1 \circ f \circ \phi_2$ for suitable $\phi_1$ and $\phi_2$.